A METHOD OF ANALYSIS OF SUBSYNCHRONOUS RESONANCE

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Title:

A method of analysis of subsynchronous resonance between large turbogenerators and the grid, intended for network planning.

Summary:

This report describes a method of analysis of the risk of subsynchronous resonance, and in particular of torsional interaction, between large turbogenerators and the grid. The method is used in the Network Planning Section for planning extensions to the high-voltage grid.

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1. Introduction

This report presents a method for the study of subsynchronous resonance, and in particular of torsional interaction, between the grid and turbogenerators, as developed by the Planning Department. The method is particularly suitable for investigation of the effects of various operational modes of the grid, and is therefore an excellent tool in network planning. One of the uses to which it has been put is investigation of the resonance conditions of the Forsmark sets, which led to work being carried out on the grid. It is therefore important that knowledge of the method should be disseminated to a wider public.

2. General

2.1 The phenomenon of subsynchronous resonance

Two failures occurred in 1970-71 in the Mohave power station in southern California in the USA. Both cases involved an insulation failure between the exciter slipring and the shaft between the generator and exciter, due to excessive heating of the shaft. This heating was caused by torsional oscillations in the whole shaft system, which in turn was due to unfortunate resonance with the series-compensated grid. This phenomenon is known as subsynchronous resonance (SSR), and considerable work has been devoted to its study throughout the world since these two breakdowns.

SSR is an umbrella name for three different phenomena, all having the common feature of electrical oscillation at a frequency in the range 0-50 Hz on the transmission grid, and involving a generator. These phenomena are described on the next page.

<u>The asynchronous effect</u> (Induction Generator Effect). As the generators behave as asynchronous generators towards a subsynchronous resonance frequency on the grid, negative resistance is supplied. If this resistance is greater than the positive resistance of the grid, the resultant resistance can be negative, causing self-induced electrical oscillation to arise in the grid.

<u>Torsional interaction</u>. If the shaft of a turbogenerator unit is made to oscillate in torsion, it will oscillate at certain frequencies, of which some are subsynchronous (0-50 Hz). If there happens to be an undesirable relationship between any of these frequencies and the resonance frequencies of the grid, torsional movements in the turbogenerator shaft line can be amplified to such a degree that damage is caused to the shaft.

<u>Transient shaft stresses</u> (Shaft Torque Amplification). Switching sequences close to the turbogenerator unit can subject it to high instantaneous stresses. Resonances in the grid can result in these stresses becoming many times greater.

This Report deals only with torsional interaction. There is no risk of induction-generator-induced oscillation on the Swedish grid, and shaft torque amplification is dealt with in other contexts.

2.2 Methods of investigating subsynchronous resonance

Many different methods of investigating SSR have been developed. Three methods, used by the Swedish State Power Board, are described overleaf.

The analytical method - the FRERED computer program, which is described in this report. It is based on analysis of the impedance conditions in the grid and generator.

Simulation. In this case, the time sequence is simulated through careful representation of the generator and turbine. This method is particularly suitable for investigation of shaft stresses, but analysis of torsional interaction using this method requires excessive computer time. An example of such a modelling program is ASEA's MOSTA program, and the EMTP program used at the Board.

Eigen value calculation. A system of equations for the turbine-generator unit and the grid can be established, using careful representation of the generator and turbine. The natural frequency of the system can be calculated from these equations, i.e. the resonance frequency with associated damping. This method is of value in investigation of torsional interaction. The calculations can be run on ASEA's MOSTA program.

2.3 Definitions and nomenclature

There has been considerable confusion in terms of nomenclature relating to sub-synchronous resonance. For example, the term Sub-Synchronous Resonance has been used without any form of discrimination for both torsional interaction, transient moment and purely electrical natural oscillations. An attempt to bring order into the somewhat chaotic terminology has been made in Reference 1, the recommendations of which have been used as far as possible in this report.

The term 'Undamped' may need further elaboration. In this context, an Undamped Sequence refers to a sequence having constant amplitude. Increasing amplitude is indicated by use of the term 'Negatively Damped' or 'Amplified'.

3. Theory

3.1 Torsional oscillations and mechanical damping

A modern turbogenerator unit has a very long shaft, linking and carrying several turbines, a generator, exciter, shaft couplings and bearings, all rotating at high speed. Any change or disturbance will cause torsional oscillations between different parts of the shaft. When analysing these torsional motions, the shaft string is usually represented by a model consisting of disc-shaped elements linked by torsion springs. The more accurately this model is to represent all torsion frequencies, the greater the number of masses which are required. This necessitates dividing the larger masses (the turbines and generator) into several smaller masses, in order to be able to represent torsional processes within them.

At the frequencies which are of interest in SSR contexts ($<$ 50 or $<$60 Hz), however, it is the behaviour of these large masses as units which is wholly predominant, and the smaller masses (bearings, couplings) can be neglected or integrated into the larger masses, as can torsional effects within the larger masses. This means that, for these frequencies, representation by a few masses is sufficiently accurate, provided that their data is modified to suit the more detailed model.

Figure 3.1 shows the schematic appearance of one of the turbogenerator units in Forsmark, and Figure 3.2 shows the corresponding mass/spring model, with representation of the moments of inertia of the masses, the inter-mass spring constants and the mechanical damping constants for the masses and springs.

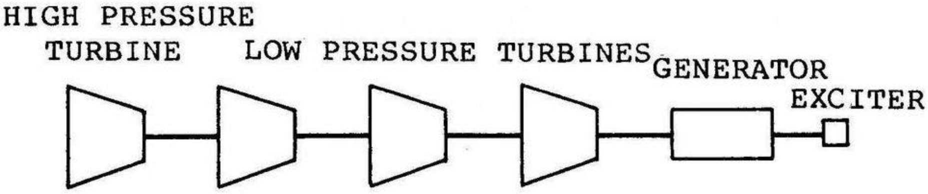


Figure 3-1 A Forsmark turbogenerator unit

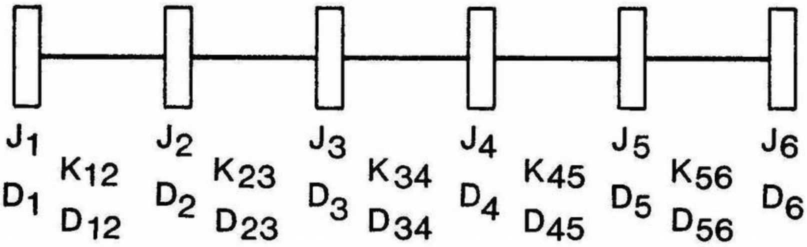


Figure 3-2 Mass/spring model

A system of this type, consisting of N masses, has (N-1) fundamental frequencies of oscillation. At each of these frequencies, the masses oscillate in relation to each other in a defined manner in terms of amplitude and phase angle. This gives rise to the concepts of mode frequency and mode form. Every torsional motion in the set can be defined in terms of a linear combination of the various mode forms.

The mode form for one of the fundamental frequencies can be described by means of a diagram showing the greatest relative angular amplitude for each mass, as shown in Figure 3-3. All the masses oscillate in phase with each other, i.e. they reach their amplitudes practically simultaneously. A mode diagram can be drawn for each natural frequency. If the points in the diagram are linked by straight lines, one line will cut the zero axis for the first torsional mode, and N lines will cut it for the Nth mode. Thus, Figure 3-3 shows the third mode. A complete mode diagram (for five modes) is shown in Appendix 1.

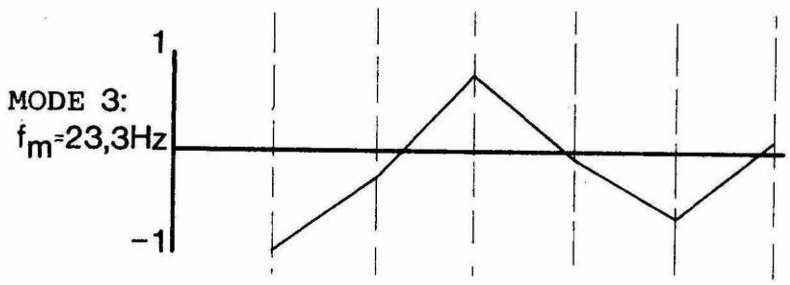


Figure 3-3. Mode form for the third mode

The advantage of the torsion modes is that it is possible, for a given mode frequency, to represent the whole shaft line as seen from a given reference point (which can suitably be the generator rotor), by a single mass and spring as shown in Figure 3-4.

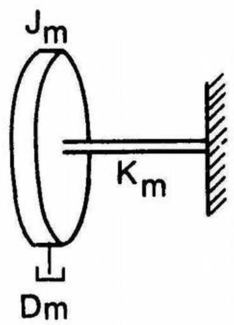


Figure 3-4. Mode model

The equation of motion for this model is

$$J_m \cdot \ddot{\varphi} + D_m \cdot \dot{\varphi} + K_m \cdot \varphi = 0$$

where φ is the mechanical angle. Knowing the mode frequency and the mode diagram, the parameters in this equation can be calculated, as shown in Appendix 2.

Damping can be expressed in several ways. D_m is known as the mode damping, and is referred to the mechanical side of the generator. Its units are [Nm/(rad/s)]. If it is referred to the rated torque and synchronous electrical angular velocity ω_0 of the generator, and called D_r, we obtain:

$$D_r = D_m \cdot \omega_o / (S_n/\omega_o) = D_m \cdot \omega_o^2/S_n$$

Other expressions for damping can be obtained directly from the equation of motion:

Damping factor $\sigma = \dfrac{D_m}{2 \cdot J_m}$ $[s^{-1}]$

Logarithmic decrement $\delta = \sigma / f_m$ [-]

Time constant $\tau = 1/\sigma$ [s]

The function D_r is used in the remainder of this analysis to indicate mechanical mode damping, as it is directly comparable with the damping effects in the electrical network.

3.2 Electrical resonance

Natural resonance circuits exist in any mesh power network containing series capacitors, and can occur as both series and parallel resonance circuits.

In the series resonance model, there is an inductance ωL in series with a capacitance $1/\omega C$ and a resistance R, as shown in Figure 3-5. When, at some given frequency, ω, the two reactances ωL and $1/\omega C$ are equal, there is a resonant condition and the impedance of the circuit falls to a very low value (R+j0). If $R \leqslant 0$, oscillation will be undamped, or increasing.

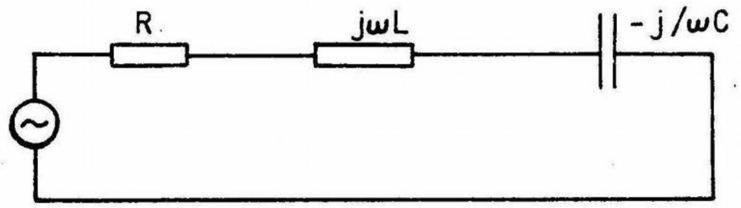


Figure 3-5. Series resonance circuit

A parallel resonance circuit can be considered as containing capacitive reactance $1/\omega C$ in one arm, and inductive reactance ωL in series with resistance R in the other arm, as shown in Figure 3-6. This presents a high impedance in the form of high resistance at resonance.

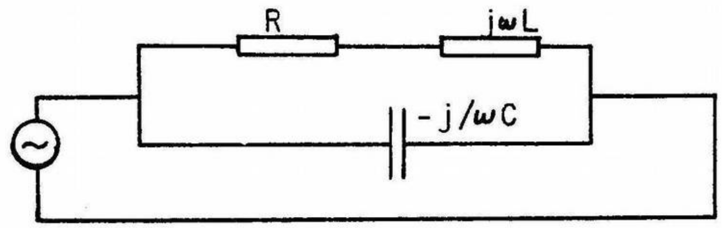
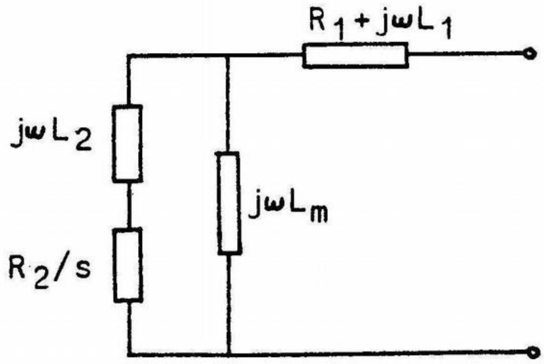


Figure 3-6. Parallel resonant circuit

If a subsynchronous frequency f_e is present on the power system, the synchronous generators on the system will lag this frequency by $(f_e - f_o)/f_e$, where f_o is 50 (or 60) Hz. As this will be a negative lag, the machines will act as induction generators with respect to frequency f_e. In simplified terms, an induction generator can be represented by a negative resistance R_2/s. The generators connected to the system thus supply negative resistance, as shown in Figure 3-7.



$$s = \frac{\omega - \omega_r}{\omega}$$
$$\omega < 2\pi \cdot 50 = \omega_r$$

Figure 3-7. Equivalent asynchronous diagram of a generator

If a tuned circuit is in resonance at frequency f_e, and if the negative resistance contribution from the generator exceeds the positive resistance of the network, undamped oscillations will arise. This is what is known as electrical resonance. Note that only the electrical functions of the generators participate in this resonance, and no sub-synchronous motions in the turbogenerator shafts are assumed.

The high transmission resistances on the Swedish grid system in relation to the installed generating capacity mean that there is no risk of purely electrical resonance with present system configuration.

3.3 Torsional interaction

So far, we have described how the shaft motions of a turbo-generator subject to subsynchronous torsional oscillation can be represented, and how electrical resonance circuits arise in a power system. Both these types of oscillation are normally damped, even if the damping effect on a turbine-generator shaft line is small. However, under certain unfortunate conditions of interaction between these two forms of oscillation, it is possible for negatively damped oscillation to arise, in which both the mechanical oscillation of the turbine-generator shaft line and the electrical oscillation in the power system interact. This is known as torsional interaction. An explanation for this phenomenon is given below.

Assume that a rotor rotates at a velocity $\omega_o = 2\pi \cdot f_o$. Superimposed on this is an angular oscillation of the rotor (Figure 3-8):

$$\theta = A \sin \mu \cdot t, \quad \text{where} \quad \mu = 2\pi \cdot f_m$$

The superimposed velocity variation is:

$$\Delta\omega = A \cdot \mu \cos \mu \cdot t$$

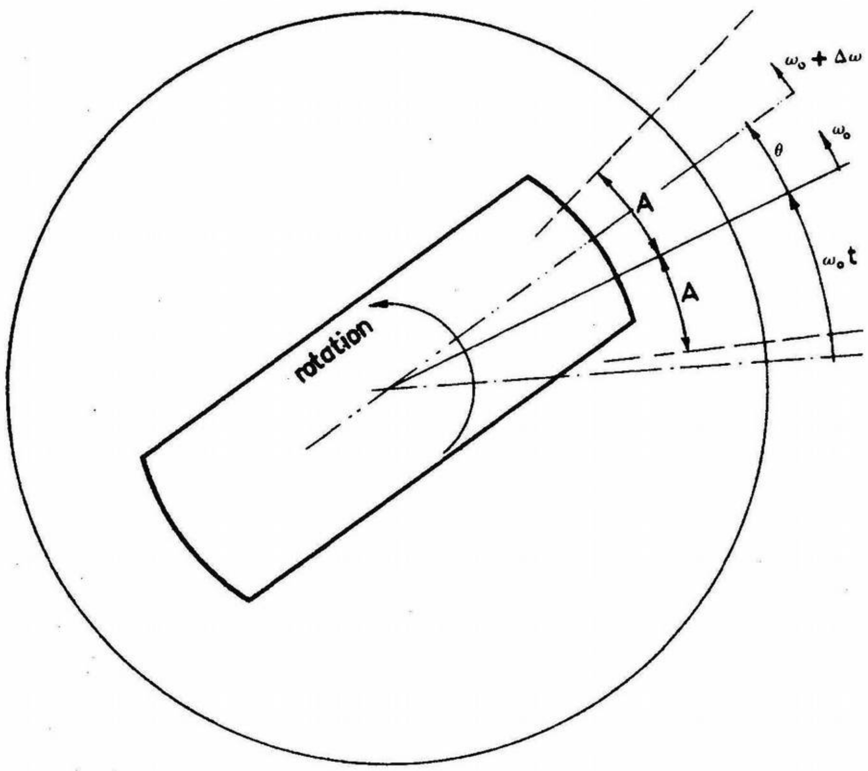


Figure 3-8. A rotating rotor with superimposed oscillation

The oscillations can be described in a dq diagram, which is regarded as rotating at synchronous speed ω_o. A is normally very small ($< 1°$).

It is assumed that the rotor and rotor flux coincide in direction d', as shown in Figure 3-9.

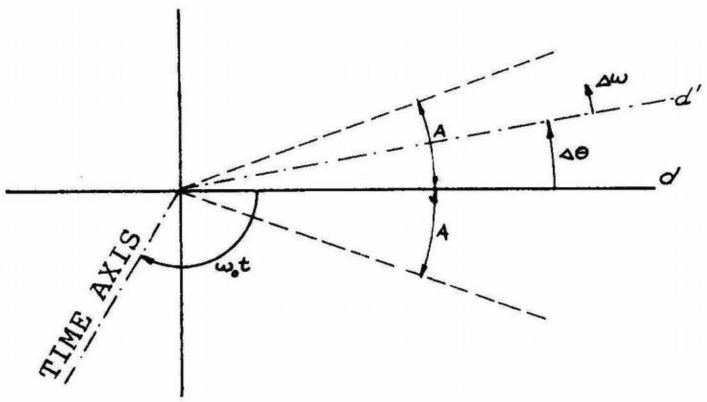


Figure 3-9. An oscillating rotor

Oscillation of the rotor field can be represented by three components (if A is small) as shown in Figure 3-10:

Φ_d rotating at ω_o, or having a constant direction along the direct axis in the dq diagram:

Φ_1 rotating at subsynchronous velocity ($\omega_o - \mu$), or with angular velocity $-\mu$ in the dq diagram:

Φ_2 rotating at supersynchronous velocity ($\omega_o + \mu$), or at angular velocity $+\mu$ in the dq diagram.

$$\Phi_d \approx \Phi_o; \quad |\Phi_1| = |\Phi_2| = \frac{A \cdot \Phi_o}{2}, \quad \text{i.e. } \Phi_1 \text{ and } \Phi_2 \ll \Phi_o.$$

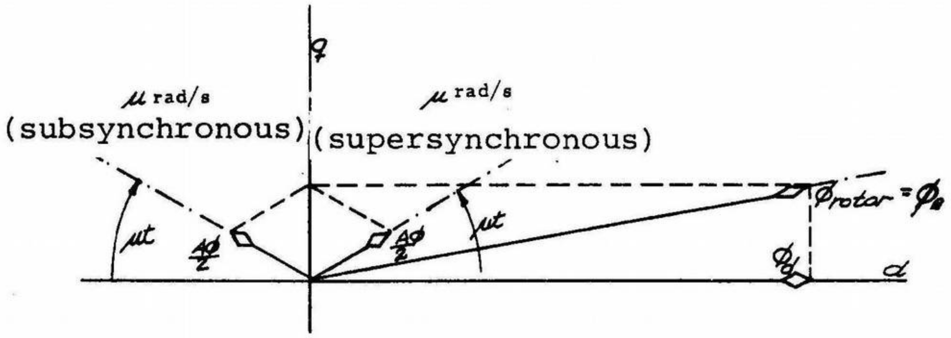


Figure 3-10. Field flux components

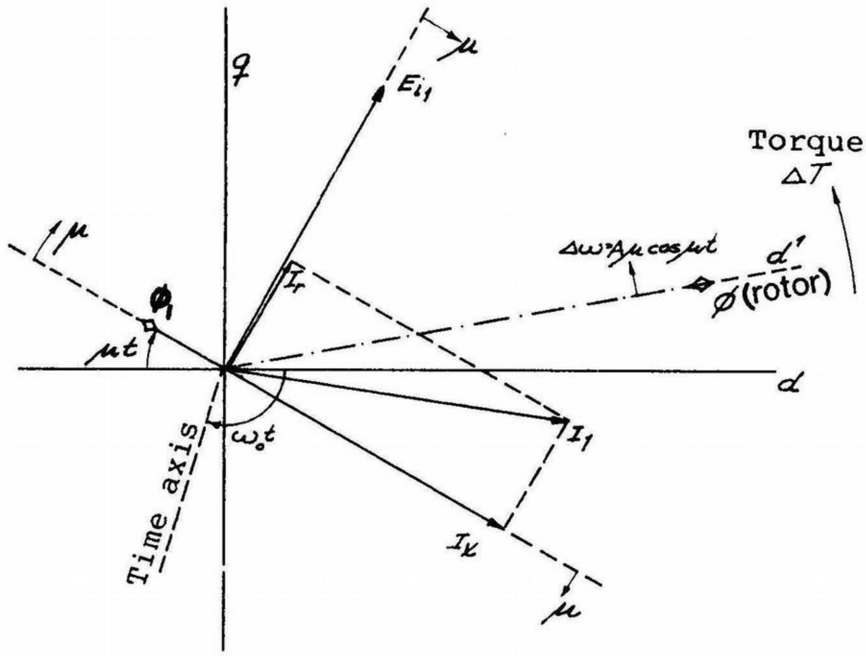


Figure 3-11. Derivation of subsynchronous torque

The <u>subsynchronous</u> flux Φ_1 induces a voltage E_{i1}. This voltage, E_{i1}, rotates with Φ_1 in the dq diagram. In principle, the phase of E_{i1} lags the flux by $\pi/2$. Bearing in mind how the time axis rotates in the dq diagram, the direction of E_{i1} can be obtained as shown in Figure 3-11.

Voltage E_{i1} gives rise to a current I_1 in the grid. The phase and amplitude of this current are determined by the impedance of the grid network (including the impedance of the generator) at the particular angular frequency $(\omega_o = \mu)$ (corresponding to frequency $[f_o - f_m]$) concerned. Similarly, the supersynchronous flux gives rise to a voltage E_{i2} and current I_2.

Both these currents will interact with the rotor flux and give rise to instantaneous contributions which can either amplify or damp the mechanical oscillations. A derivation of the magnitude and effect of these contributions can be found in Reference 2. Here, we shall merely point out that if the impedance presented to voltage E_{i1} is represented by $(R + jX)$, the subsychronous current I_1 will give a damping contribution D_1.

$$D_1 = -\frac{(\omega_o - \mu)}{2} \cdot \frac{R}{R^2 + X^2} = -\frac{f_1}{2f_m} \cdot \frac{R}{R^2 + X^2}$$

where $f_1 = (f_o - f_m)$ is the frequency for which R and X are to be calculated.

Similarly, the supersynchronous current I_2 gives a damping contribution D_2:

$$D_2 = \frac{(\omega_o + \mu)}{2} \cdot \frac{R}{R^2 + X^2} = \frac{f_2}{2f_m} \cdot \frac{R}{R^2 + X^2}$$

where $f_2 = (f_o + f_m)$.

Summarising, it can be said that:

- A rotor which oscillates at a frequency f_m gives rise to a subsynchronous current of frequency $f_1 = (f_o - f_m)$ and a supersynchronous current of frequency $f_2 = (f_o + f_m)$.
- These currents interact with the rotor flux to produce a damping contribution.
- These damping contributions can be indicated by:

$$D_1 = - \frac{f_1}{2f_m} \cdot \frac{R}{R^2 + X^2}$$

$$D_2 = \frac{f_2}{2f_m} \cdot \frac{R}{R^2 + X^2}$$

where R and X are calculated for frequencies f_1 and f_2 respectively.

Using these functions, damping contributions D_1 and D_2 from torsional interaction can be directly compared with the purely mechanical damping D_r. Note that as long as R is positive, D_1 gives a negative damping contribution. D_2 is always positive, as R never becomes negative for supersynchronous frequencies.

The determining criterion for avoiding any risk of SSR can thus be formulated as:

$$\underline{\underline{D_r + D_1 + D_2 > 0}}$$

4. Method of analysis

The method of analysis for determining the risk of torsional interaction which has been developed by the Board is directly based on the simplified theory for the origin of this interaction, described above. The basis for the method has been obtained from References 2 and 13. Although particularly simple in mathematical terms, it shows good agreement with the results from more advanced methods. Its major advantages are:

- simple calculations result in a minimum of computer time, and permit the power network to be represented by a quite large model network:
- a large number of different operational modes and load cases can be investigated quite simply:
- the method provides a feel for the resonance behaviour of the network, which can be applied in assessing the behaviour of other cases, and
- damping from torsional interaction is calculated quite independently of mechanical damping in the turbogenerator, which is the most uncertain parameter. This means that the risks can be assessed quite simply for different levels of mechanical damping.

The principle of the method is simple. The subsynchronous (D_1) and the supersynchronous (D_2) damping contributions from torsional interaction are calculated for a certain mechanical mode frequency, f_m, and are compared with the assumed mechanical mode damping. If the condition $D_1 + D_2 + D_r > 0$ is not fulfilled, there is a risk of subsynchronous resonance.

4.1 Representation of the grid and turbogenerator

As derived above, the damping contributions from torsional interaction can be calculated from the formulae:

$$D_1 = - \frac{f_1}{2f_m} \cdot \frac{R}{R^2 + X^2} \quad \text{where } f_1 = (f_o - f_m) = (50 - f_m)$$

and

$$D_2 = \frac{f_2}{2f_m} \cdot \frac{R}{R^2 + X^2} \quad \text{where } f_2 = (f_o + f_m) = (50 + f_m$$

where f_m is the particular mode frequency concerned, and R and X are to be calculated at frequencies f_1 and f_2 respectively.

The stator emf E, varying sinusoidally at frequency f, faces the impedance (R + jX) in the power network. This also includes the impedance of the generator. Figure 4-1 shows schematically what (R + jX) represents.

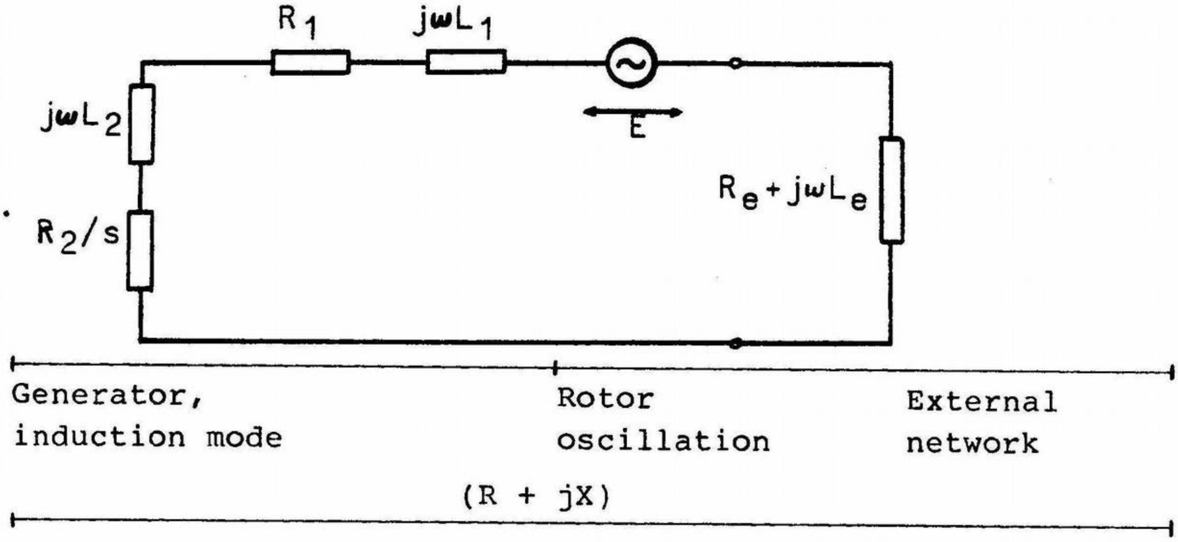


Figure 4-1. Equivalent electrical circuit

All frequency-dependent impedances must be represented with inclusion of their frequency dependence, i.e. it is essential to differentiate between inductive reactances (ωL) and capacitive reactances ($1/\omega C$).

Only a minor portion of the power grid is represented in detail in the calculations. Peripheral parts of the network and the networks at lower voltage are represented by their short-circuit impedance.

4.1.1 The generator being investigated

The generator being investigated is represented by its induction mode equivalent, as it behaves as an induction machine with respect to frequencies f_1 and f_2 with a slip of $s = (f - f_o)/f$, where $f = f_1$ or f_2 and $f_o = 50$ (or 60) Hz. In general, the data for this arrangement is not given. However, reactance $(X_1 + X_2)$ can be expressed as $(X_d'' + X_q'')/2$ i.e. the average value of the direct-axis and quadrature-axis subtransient reactances. X_m and R_1 can be neglected, and R_2 can be assumed to lie between 1-3% for large machines (References 3 and 4).

4.1.2 Other generators and short-circuit impedances

Generators which are connected directly to the model network should be represented by their induction machine equivalents, while other generators should be represented by short-circuit impedances at the nodes of the model network.

These short-circuit impedances must not be set at the maximum 'externally acting' short-circuit powers at the nodes, as this will give altogether too low a value of impedance.

Instead, a network calculation program should be used to calculate them for some given production situation, ensuring that the whole power network is adequately represented. It is then the short-circuit power at each node, with simultaneous short-circuiting of all the model network nodes which is of interest.

4.1.3 The Network

Lines and transformers on the distribution system can be represented by their π links, i.e. with two $1/\omega C$ shunt elements and an $(R + j\omega L)$ series element. However, the shunt elements have so little effect that they are often ignored.

4.1.4 Loads

System loads can be represented by shunt elements. If most of the load consists of large motors, representation in the form of short-circuit power is more relevant. However, compared with the short-circuit impedances which represent the rest of the network, the impedance of the load is quite high, and so it can often be ignored in the calculations. This applies, too, to the internal load of the turbogenerator under investigation.

4.1.5 The mechanical parts of the turbogenerator

The mechanical parts of the turbogenerator being investigated are represented only by their mode frequencies and associated mechanical mode dampings.

4.2 Method of calculation

The calculations are performed by a computer program known as FRERED (from Frequency and Reduction). The central calculation consists of reducing the impedance of the model network to a single equivalent impedance, as seen by the generator. This impedance must include the impedance of the generator itself. The reduction is performed at the subsynchronous frequency f_1 (= $f_o - f_m$) and at the supersynchronous frequency f_2 (= $f_o + f_m$). From the equivalent impedances, the subsynchronous and supersynchronous damping contributions can then be calculated, and compared with the mode damping for mode frequency f_m.

This procedure should then be repeated for all mode frequencies which are suspected of being dangerous, and for several different operational and loading states of the distribution grid. However, certain procedures are available for simplifying calculation, and are described below.

4.2.1 Variation of frequency

The disadvantages of the method of calculating damping contributions for certain given fixed mechanical mode frequencies are that the calculation must be performed for each mode frequency which is suspect, and that the damping effect is quantified only at exactly that frequency. Inaccuracies in input data may result in the real mode frequency differing somewhat from the calculated mode frequency.

A better method than such specific point calculation is to calculate the contribution for a group of frequencies around the frequencies of interest, or for frequencies in the whole subsynchronous and supersynchronous ranges. This enables curves for the magnitude of the damping contributions as a function of mechanical mode frequency to be drawn.

Figure 4-2 shows such curves for the subsynchronous contribution (D_1), the supersynchronous contribution (D_2) and the total damping contribution from torsional interaction ($D_1 + D_2$). Note that the scales are dissimilar: a mechanical frequency variation in the 10-40 Hz range corresponds, for instance, to a subsynchronous frequency of 40-10 Hz and a supersynchronous frequency of 60-90 Hz.

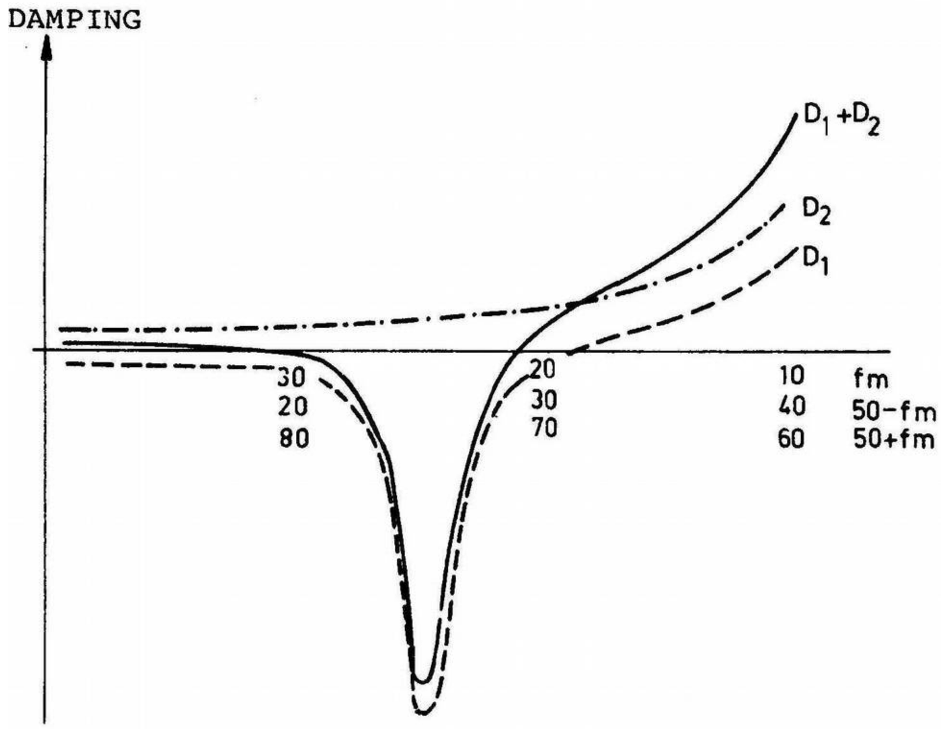


Figure 4-2. Subsynchronous and supersynchronous damping contributions

The curve of $(D_1 + D_2)$ can be interpreted as follows. If the turbogenerator has a mode frequency at f_m, the mechanical mode damping D_r for this mode must be sufficiently large to ensure that the sum of $D_1 + D_2 + D_r > 0$. If the set has a mode at $f_m = 25$ Hz, D_r must therefore be greater than the amplitude of the resonance peak.

This frequency scanning method enables the damping at all mode frequencies, and the sensitivity to errors in calculation of the mode frequencies, to be estimated. It also provides a picture of the resonance conditions in the network. Changes in the network structure and load level will affect the appearance of the curves in a systematic manner.

It might appear that the calculations could be further simplified by performing joint calculation of D_1 and D_2, and thus calculating the sum ($D_1 + D_2$) as a function of the corresponding mechanical frequency f_m. However, this is unsuitable for several reasons: partly because it would be less clear which electrical frequencies occur on the grid, and partly because less extensive calculations are needed for D_2 than for D_1. This is because the magnitude of D_2 is small, and relatively constant for (smaller) changes in the grid. This means that if D_2 is calculated for a basic case in some particular investigation, the values obtained can be regarded as applicable to the whole investigation. D_r and D_2 can be added for the various modes, enabling the risk of occurrence of subsynchronous resonance to be expressed as:

$$D_1 < - (D_2 + D_r)$$

D_2 will not be considered in much detail in the rest of this report.

4.2.2 Variations of network parameters

Another way of obtaining a picture of the resonance behaviour of the distribution system is to investigate the damping for varying values of some network parameter at a given electrical frequency f, corresponding to a mode frequency, f_m. Among the network parameters which can be of interest in this application are the degree of compensation of series-compensated lines, which can assume different values depending on the number of series-connected compensator banks which are connected (in Sweden: two or three compensation banks in series).

The results of variation of the size of a series capacitor in this manner are shown in Figure 4-3. D_1 has been calculated for 0-100% compensation for the reactance of the compensated line, and at two frequencies, corresponding to two mechanical oscillation modes.

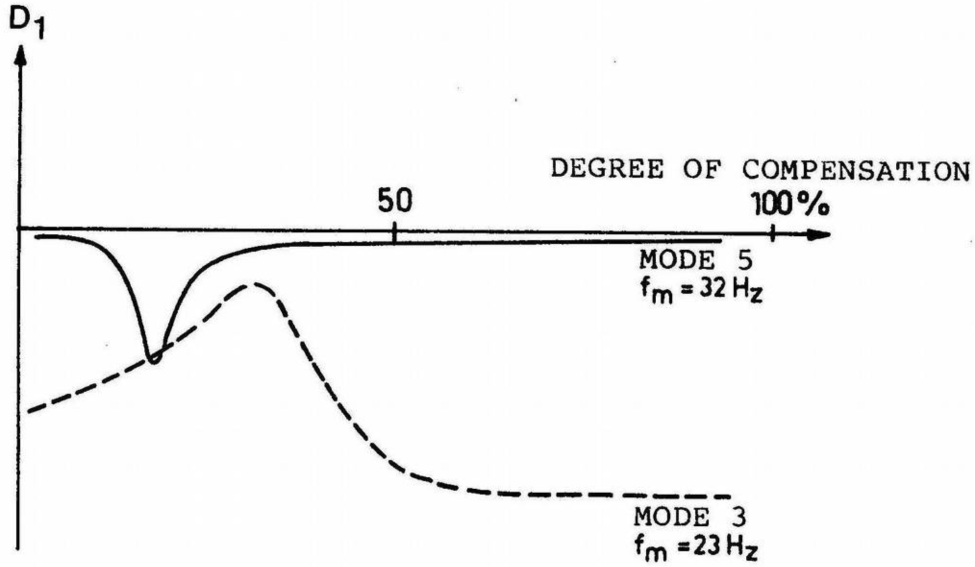


Figure 4-3. D_1 for various degrees of compensation

If several series-compensated lines are connected near to the turbogenerator set being investigated, the resonance picture can become quite complicated. Nevertheless, one method of obtaining a grasp of it can be to calculate D_1 as a function of the compensation on two different lines for a given mode. The result can be expressed in the form of 'level contours' in a diagram, as shown in Figure 4-4.

Note that this diagram only shows the resonance conditions for one mode frequency. Although, from Figure 4-4, it seems as if very high compensation could be successfully used, it is often found that some other mode frequency will run into trouble. This also applies for lesser degrees of compensation.

Figure 4-5 shows the conditions for a mode at a higher mode frequency. If the mechanical damping D_r does not compensate for these negative dampings, only a small amount of compensation is permissible.

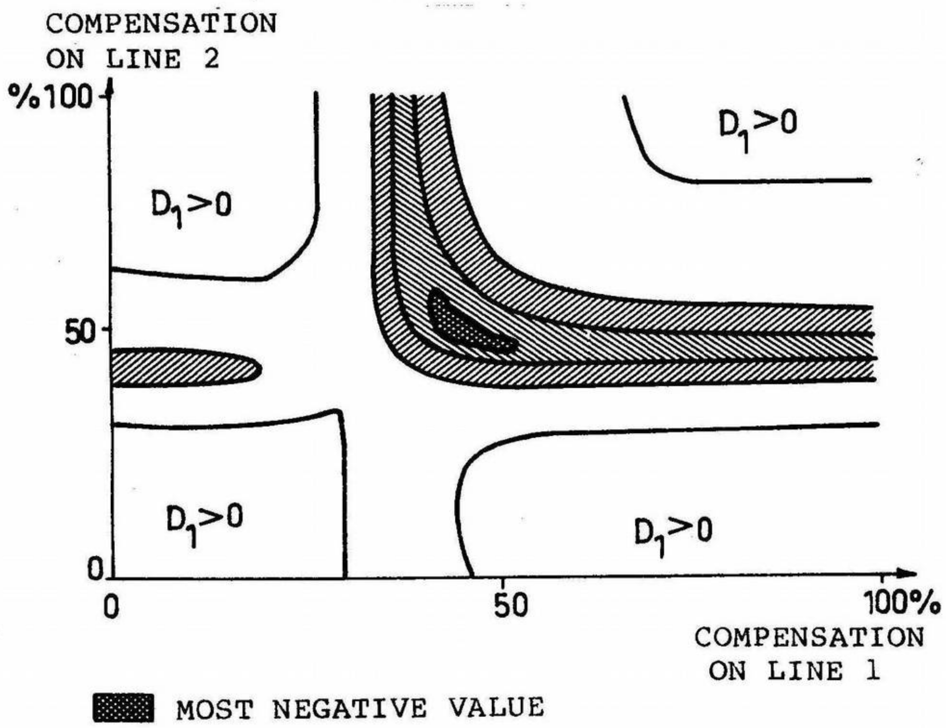


Figure 4-4. D_1 as a function of compensation on two lines

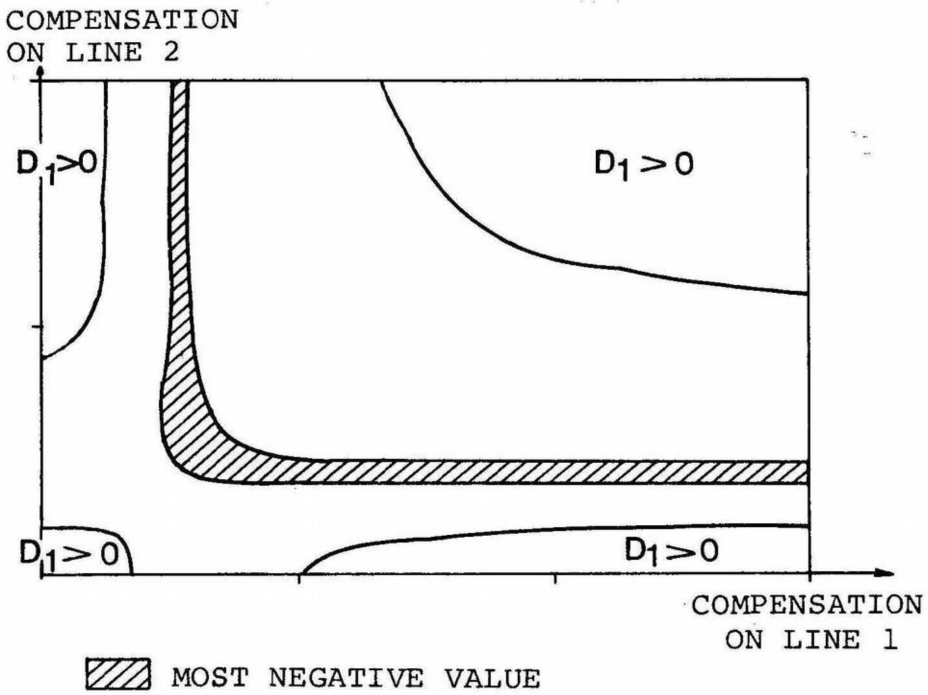


Figure 4-5. D_1 as a function of compensation on two lines

4.2.3 Comparison of resistance

So far, this analysis has concentrated on investigation of the variation in D_1. A lot can be learned about the power network by investigating the calculated equivalent impedance, $(R + jX)$. This can be plotted as a function of the frequency (Figure 4-6) in the same way as for D_1.

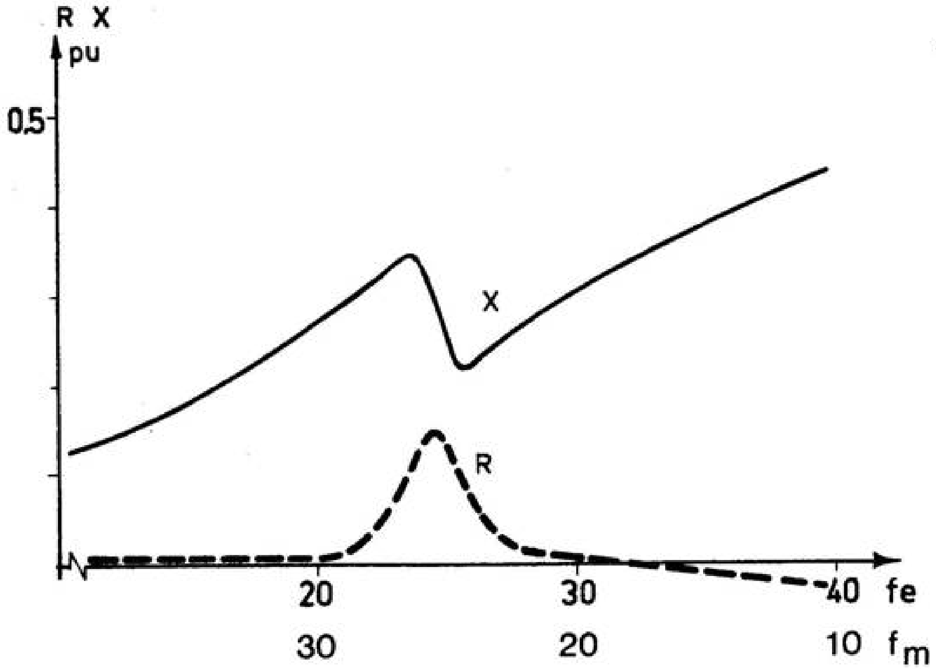


Figure 4-6. Equivalent impedance at varying frequency

Resistance is an interesting factor. In the same way as impedance can be converted to damping, mechanical damping can also be converted to equivalent resistance, which can be compared with the value calculated from the power network.

For subsynchronous frequency, we have:

$$D_1 = - \frac{f_1}{2f_m} \cdot \frac{R}{R^2 + X^2}$$

If the supersynchronous contribution is ignored, the necessary condition for stability is:

$$D_r > - D_1$$

If a 'damping resistance' R_m, corresponding to D_r, is defined as:

$$R_m = \frac{f_1}{2f_m} \cdot \frac{1}{D_r}$$

we obtain:

$$\frac{f_1}{2f_m} \cdot \frac{1}{R_m} > \frac{f_1}{2f_m} \cdot \frac{R}{R^2 + X^2}$$

which can be developed to:

$$R^2 + X^2 > R \cdot R_m$$

or

$$(R - \frac{R_m}{2})^2 + X^2 > (\frac{R_m}{2})^2$$

This corresponds to a circle in an R-X diagram, having its centre at $R = R_m/2$, $X = 0$ and of radius $R_m/2$. In order to avoid subsynchronous resonance, R and X should lie outside the circle. However, it order to avoid natural electrical resonance, $R > 0$. Figure 4-7 shows the appearance of the circle.

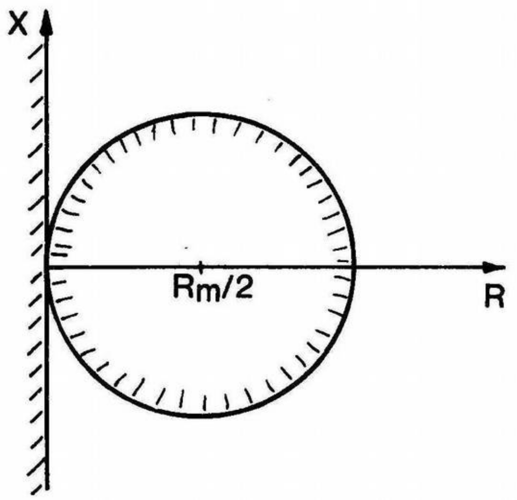


Figure 4-7. Forbidden area for R, X

If the calculated point R,X lands inside the circle, Figure 4-7 can be used to read off the necessary resistance and/or reactance which must be added in order to obtain stability. This information is of value if filters are employed in order to deal with sub-synchronous resonance.

4.3 Comparison with other methods of analysis

The results obtained from the method of analysis described on the preceding pages, produced by the FRERED computer program, have been compared with other methods of analysis. The two most important comparisons are that with an IEEE Benchmark Model (a reference model for test of analytical methods) and a comparison with eigen-value calculations carried out by ASEA's MOSTA program.

IEEE: Calculations have been carried out on the IEEE network and compared with the 'answer'. Very good agreement has been observed. The calculations are presented in Reference 12.

MOSTA: The turbogenerator unit in MOSTA is represented by a spring/mass model. The generator is represented in detail. This program was originally intended to be a simulation program, but it can also be used to calculate eigen values for the system under investigation. With certain assumptions of turbine damping, it is then possible to determine the damping at all natural frequencies in both the electrical and mechanical systems.

Calculations have been made for Forsmark using both FRERED and MOSTA. Appendix 3 shows comparison of some of the results, from which the good agreement can be seen.

4.4 Parameter discussion

The above comparisons have enabled the validity of the analytical method to be established. Having done so, the success of any analysis depends upon proper representation of the power system and on the accuracy of the data used.

Network representation is, in principle, similar for all analytical methods. It is important that all nearby series-compensated lines are represented, and that the short-circuit impedances at the nodes of the model network which represent the surrounding and basic system have been properly calculated. In general, the value which is obtained if the maximum 'externally acting' short-circuit power at the node is assumed is too high. It is preferable, starting from a detailed load distribution calculation on a network calculation program, to calculate the contribution at each node of resulting from simultaneous short-circuiting of all nodes of interest in the model network. This should be done for all load and production cases to be investigated (low load, high load, different operating conditions etc.).

The effects of load, when load is represented by a shunt impedance, are generally quite small. However, the contribution which the load can make to short-circuit power can be of interest, although this contribution is not obtained from the calculation described in the preceding paragraph.

Generator representation in FRERED is very simple in comparison with the majority of other methods of analysis. However, it can be difficult to arrive at accurate data for this simple representation. A discussion can be found in References 3 and 4. A reactance value of $X = (X_d'' + X_q'')/2$ and a resistance value of R_2 = 1-3% are standard values.

Turbine representation is based on mode frequency and mode damping. When comparing calculated and measured values of mode frequency, an accuracy of ±1 Hz can generally be assumed (Reference 3). For mode damping, however, the accuracy is not quite so good. Mode damping is obtained from the assumed damping in turbines and shafts. In general, damping in shafts can be ignored in comparison with damping in the turbines. This latter effect arises primarily from interaction between the turbine blades and the steam, and thus varies with the load on the machine. It is least at no-load, and then increases rapidly with increasing load.

Damping in the turbines is particularly difficult to calculate. Measurements and calculations have been made in other countries, but the results vary quite considerably depending on the design of the turbines.

Until more accurate values can be obtained, pessimistic values must be assumed for the purposes of these investigations. Appendix 4 is a presentation of a number of results of measurements and calculations which have been made in other countries, and of the values of damping assumed by the Board. Section 3.1 explains the units which have been used.

5. Measures against sub-synchronous resonance

References 5 and 6 provide a good survey of various methods of countering sub-synchronous resonance and of protecting turbogenerator units against it. Nevertheless, a brief discussion of the various ways open can be in place here. Only measures intended to counter torsional interaction are included: there are further means of protection available against transient torques. Countermeasures can be divided up into two types: those which operate continuously, e.g. those which apply damping to the system, and those which act first when subsynchronous resonance occurs. Examples of the first type of measure are:

- altered network configuration (new lines, new routes, removal of lines from service):
- alteration of network series compensation:
- filters of various types (tuned to the resonance frequencies) in association with generators and/or series capacitors:
- thyristor-controlled shunt reactors close to generators, intended to control torsional oscillation of the rotor:
- additional facilities in the generator voltage regulator, AVR (Speed Error Deviation Control, SEDC), and
- special operating routines.

Various types of relay protection are used to protect the turbogenerator unit if subsynchronous resonance actually occurs:

- Measurement of shaft movement of the set. Subsynchronous resonance causes the set to be tripped or series capacitors to be shunted out.

- Measurement of the subsynchronous component in the generator output current. Subsynchronous resonance causes the set to be tripped or series capacitors to be shunted out.

- Measurement of the subsynchronous component in series capacitor current. Subsynchronous resonance causes the capacitor to be shunted out.

Measures have been applied to the Swedish grid to counter the risk of subsynchronous resonance at Forsmark. They consist of permanent bypassing of a series capacitor, restructuring of fault sectioning and special operating instructions. However, some of these measures must be regarded as provisional. For the future, other measures must be taken, e.g. relay protection to provide protection against more unlikely situations.

One of the projects (Project 32) included in the Swedish State Power Board/ASEA joint development work involves the study of countermeasures against subsynchronous resonance.

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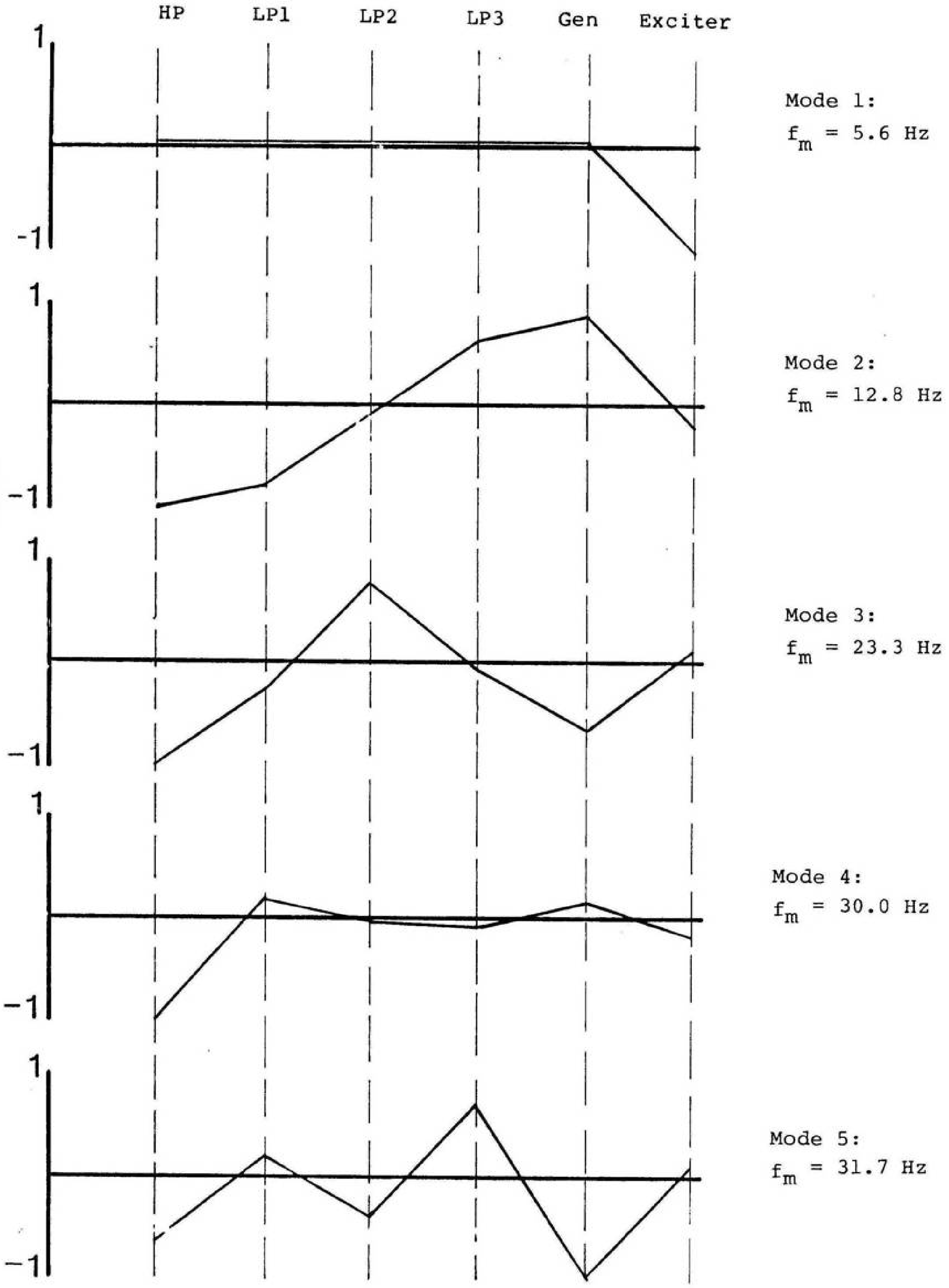
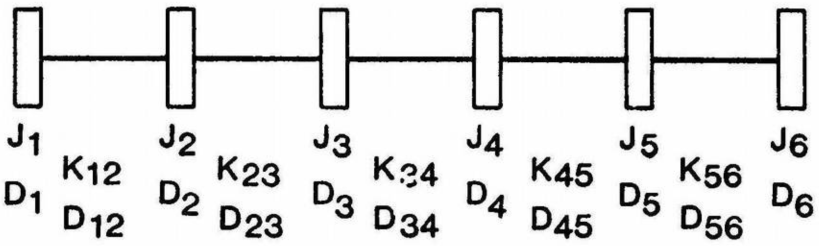
FORSMARK 1 AND 2

MODE FREQUENCIES AND MODE FORMS



MODE DIAGRAM AND MODE DAMPING



J:	Moment of inertia	[kgm^2]
K:	Spring constant	[Nm/rad]
D:	Damping	[Nm/(rad/s)]

Figure 1. Spring/mass model

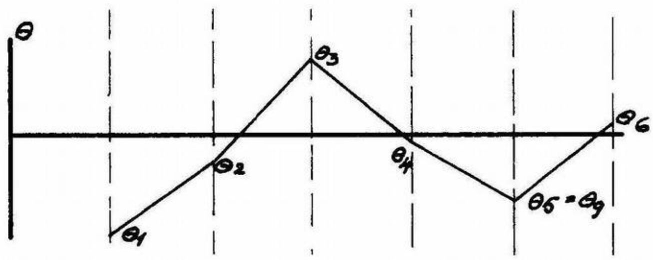


Figure 2. Mode form for mode frequency f_m

Assume that a turbine-generator unit is represented by the spring/mass model as shown in Figure 1, and has a mode diagram for mode frequency f_m as shown in Figure 2. Seen from the generator (mass 5), the behaviour of the shaft at a mode frequency of f_m can be symbolised by a single mass and spring as shown in Figure 3.

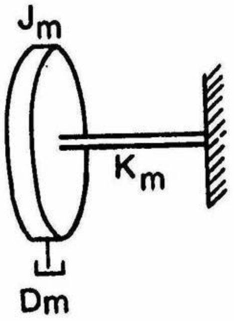


Figure 3. Mode model

This mode model is characterised by the functions:

D_m mode damping
J_m mode moment of inertia
K_m mode spring constant

which are elements in the equation of motion for the mode:

$$J_m \cdot \ddot{\varphi} + D_m \cdot \dot{\varphi} + K_m \cdot \varphi = 0, \qquad (1)$$

where φ is the mechanical angle of the rotor.

This equation can be rewritten to the well-known equation

$$\ddot{\varphi} + 2\sigma \cdot \dot{\varphi} + \omega_m^2 \cdot \varphi = 0; \quad 2\sigma = D_m/J_m; \quad \omega_m^2 = K_m/J_m$$

where σ is called the damping factor or damping constant, and ω_m is the fundamental angular frequency of the undamped system. The solution to this equation is (if $\sigma < \omega_m$):

$$\varphi = A \cdot e^{-\sigma \cdot t} \cdot \cos(\omega t + \varphi_o) \qquad (2)$$

where $\omega = \sqrt{\omega_m^2 - \sigma^2}$. For turbogenerator sets, $\sigma \ll \omega_m$, which means that it is acceptably accurate to say that $\omega = \omega_m$, i.e. that the oscillation frequency (the mode frequency) is not affected by the damping.

Parameters J_m, D_m and K_m, included in the equation of motion, can be calculated from:

$$J_m = (J_1 \cdot \theta_1^2 + J_2 \cdot \theta_2^2 + \ldots)/\theta_g^2$$

$$D_m = (D_1 \cdot \theta_1^2 + D_2 \cdot \theta_2^2 + \ldots)/\theta_g^2 + (D_{12} \cdot (\theta_1 - \theta_2)^2 + D_{23} \cdot (\theta_2 - \theta_3)^2 + \ldots)/\theta_g^2$$

$$K_m = J_m \cdot \omega_m^2.$$

Of these, the function which is of most interest in the context of sybsynchronous resonance is mode damping, D_m. In the equation, its dimensions are [Nm/(rad/s)], where rad/s relates to the mechanical rotation of the shaft. In order to be able to compare this with damping in the electrical system, it must be converted to electrical angular velocity. This can be done by expressing it as a function of the generator rated torque and synchronous angular velocity.

Rated torque $M_{vn} = S_n/\omega_o$, where S_n is the rated power of the generator.

Synchronous angular velocity $\omega_o = \frac{2}{p} \cdot 2\pi \cdot 50$, where p = the number of generator poles.

If the standardised damping is indicated by D_r, we obtain the expression:

$$D_r = D_m \cdot \frac{\omega_o}{\frac{S_n}{\omega_o}} = D_m \cdot \frac{\omega^2}{S_n}$$

Damping can be expressed in several other ways. The damping factor σ can be derived from Equation (2) above. Its reciprocal is the time constant τ. The ratio of two successive amplitude peaks is another measure of the damping, known as the damping ratio k. The logarithm of k is known as the 'logarithmic decrement', indicated by δ. The summary of all these parameters, and their relationships, is given below.

D_m	'Absolute damping' [Nm/(rad/s)]
$D_r = D_m \cdot \omega_o^2/S_n$	'Relative damping' [p.u.torque/p.u.angular velocity/s]
$\sigma = D_m/2J_m$	Damping factor [s^{-1}]
$\tau = 1/$	Time constant [s]
$k = e^{\sigma \cdot (1/f_m)}$	Damping ratio [-]
$\delta = \ln k = \sigma/f_m$	Logarithmic decrement [-]

Physical values have been used throughout in the above description. However, data for the spring/mass model is often expressed in per unit values, referred to the generator rated data, but still using mechanical angles. It should be noted in this context that as the inertia constant H is expressed in MWs/MVA referred to the rated generated power S_n, and as other functions (below) are referred to the rated torque of the machine, the synchronous angular velocity (and thereby also the electrical angle) is hidden in these equations through the relationship $S_n = M_{vn} \cdot \omega_o$. If relative values are indicated by ', the following relationships between relative and absolute values are obtained:

$$K'_{ij} = K_{ij} \cdot \omega_o / S_n \qquad \text{[p.u. torque/(rad)]}$$

$$D'_i = D_i \cdot \omega_o / S_n \qquad \text{[p.u. torque/rad/s)]}$$

$$H'_i = \frac{1}{2} J_i \cdot \omega_o^2 / S_n \qquad \text{[MWs/MVA]}$$

The same relationships apply for mode quantities K'_m, D'_m and H'_m, and so Equation (1) transforms to:

$$2H'_m \cdot \ddot{\varphi} + D'_m \cdot \omega_o \cdot \dot{\varphi} + K'_m \cdot \omega_o \cdot \varphi = 0$$

These parameters are then obtained from:

$$H'_m = (H'_1 \cdot \theta_1^2 + H'_2 \cdot \theta_2^2 + \ldots)/\theta_g^2$$

$$D'_m = (D'_1 \cdot \theta_1^2 + D'_2 \cdot \theta_2^2 + \ldots)/\theta_g^2 + [D'_{12}(\theta_1-\theta_2)^2 + D'_{23}(\theta_2-\theta_3)^2 \ldots]/\theta_g^2$$

$$K'_m = 2 \cdot H_m \cdot \omega_m^2/\omega_o$$

The various dampings then become:

$$D_r = \omega_o \cdot D'_m$$

$$\sigma = D_r/4 \cdot H'_m$$

τ, k, and δ as before.

A COMPARISON BETWEEN FRERED AND MOSTA

MOSTA — By entering varying values of turbine damping D_T for a given system operating mode, a family of values of total damping is obtained. Linear regression then enables the value of D_T which corresponds to a total damping of zero to be calculated.

FRERED — The mechanical mode damping D_r which is necessary to produce a total damping of zero is obtained by addition of the subsynchronous damping contribution D_1 and the supersynchronous contribution D_2. As D_r is proportional to the turbine damping D_T, the value of D_T which corresponds to a total damping of zero can be calculated.

The results of two operating examples follow. In both cases, the mechanical mode frequency is 23.3 Hz. σ = total damping.

Operating mode	MOSTA D_T	MOSTA σ	FRERED
Intact grid and full machine output	0.00300	-0.16765	D_1 = - 0.44
	0.00030	-0.00049	D_2 = 0.07
	0.00015	0.00880	D_r = 0.37 for $\sigma = 0$
	0.00006	0.01436	
	0.00029	0.00000	$D_T(\sigma = 0)$ = 0.00030
X_c(CL3) = -54 ohm: otherwise, intact grid and full machine output	0.00100	-0.00803	D_1 = - 1.02
	0.00050	0.02264	D_2 = 0.07
	0.00030	0.03494	D_r = 0.95 for $\sigma = 0$
	0.00015	0.04416	
	0.00087	0.00000	$D_T(\sigma = 0)$ = 0.00078

MECHANICAL DAMPING

Mechanical damping is a particularly uncertain, although important, parameter in the investigation of torsional interaction. Most of the damping originates in the turbines. However, this turbine damping is difficult to measure, and so it is generally the mode damping for the modes of the whole turbogenerator set which is measured in tests. This is also the damping which is generally stated in reports. Although the mode damping depends on the turbine damping, it also depends on the mode form of the whole set. It is therefore preferable to compare turbine damping between different turbogenerator sets than to compare mode damping.

Reference 7 describes simulation of the turbine blade dynamics. It states the following damping for the various turbine units (P_k/P_n are ratios which express the respective contribution of each turbine to the total output power):

Turbine	D_t [MW/(MVA·rad/s)]	P_k/P_n (%)	$D_t \cdot P_n/P_k$
High-pressure	0.00076	24	0.00317
Int.-pressure	0.00108	34	0.00318
Low-pressure 1	0.00045	14	0.00321
2	0.00045	14	0.00321
3	0.00045	14	0.00321

As can be seen, the assumed damping has been distributed among the turbines in proportion to their different output powers. If the same procedure is adopted for the Forsmark sets, and the damping for a low-pressure turbine is assumed to be 0.0004, the following dampings are obtained:

Turbine	P_k/P_n	D_t
Low-pressure 1	20	0.0004
2	20	0.0004
3	20	0.0004
High-pressure	40	0.0008

This gives the following mode dampings:

Mode number	1	2	3	4	5
Mode frequency f_m [Hz]	5.6	12.8	23.3	30.0	31.7
Log. decrement δ [-]	0.0061	0.0031	0.0022	0.0072	0.0008
Damping factor σ [s^{-1}]	0.034	0.040	0.051	0.215	0.024
Time constant τ [s]	29.5	25.3	19.7	4.65	42.0

As there is little or nothing to be found in the literature in turbine damping, comparisons with other units must be based on mode dampings. A number of such comparisons are given below.

1. For the Mohave unit (483 + 426 MVA), the following measured values of damping have been stated (Reference 8):

Mode number	1	2	3
Mode frequency f_m [Hz]	26.7	30.1	56.1
Logarithmic decrement δ [-]	0.003	0.005	0.001
Damping factor σ [s^{-1}]	0.08	0.15	0.056
Time constant τ [s]	12.5	6.7	17.9

These values apply for low load. At high output, δ and σ must be multiplied by between 2 and 5.

2. For the Navajo unit (892 MVA) the following measured values have been stated (Reference 9):

Mode number	1	2	3	4
Mode frequency f_m [Hz]	15.8	20.2	26.0	33.2
Logarithmic decrement δ [-]	0.0040	0.0065	0.0015	0.0010
Damping factor σ [s^{-1}]	0.06	0.13	0.04	0.03
Time constant τ [s]	15.8	7.6	25.6	30.1

For a loaded unit, δ and σ must be multiplied by (mode number in brackets): 4.0(1); 1.85(2); 16(3); 2.5(4).

3. For the same unit, Reference 3 gives the following measured values of damping at no-load:

Mode number	1	2	3	4
Mode frequency f_m [Hz]	15.75	20.35	25.94	32.28
Logarithmic decrement δ [-]	0.0032	0.0056	0.0011	0.0009
Damping factor σ [s^{-1}]	0.050	0.113	0.028	0.028
Time constant τ [s]	20	8.9	35.7	35.7

For a loaded unit, δ and σ must be multiplied by: 5(1); 1.9(2); 19(3); 3.8(4).

4. For Cholla (321 MVA), measurements have been carried out at a resonant frequency (Reference 10):

Mode frequency f_m [Hz]	47.7
Logarithmic decrement δ [-]	0.0085
Damping factor σ [s^{-1}]	0.41
Time constant τ [s]	2.5

5. The following values of damping have been assumed in connection with investigations for the Coronado station (Reference 11):

Mode frequency f_m [Hz]	43.3
Logarithmic decrement δ [-]	0.0009
Damping factor σ [s^{-1}]	0.04
Time constant τ [s]	25